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New Approach for Modeling, Analysis, and Control of Air Traffic Flow

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An Eulerian approach to modeling air traffic flow is advanced. This modeling technique spatially aggregates air traffic to generate models of air traffic flow in a network of interconnected, one-dimensional control volumes. The approach simplifies the problem of characterizing the air traffic flow because the order of the corresponding airspace model depends only on the number of spatial control volumes used to represent the air traffic environment and not on the number of aircraft operating in it. Under a quasi-steady-state assumption, this process results in linear models of the air traffic environment. It is shown that analysis and design methods from linear control theory can be applied to this model to yield useful approaches for characterizing and controlling the air traffic flow.

I. Introduction

MODELING and analysis of the air traffic environment has been of interest to the air traffic management (ATM) community, as is evidenced by recent workshops focusing on these issues.[1,2] These research initiatives seek to address the air traffic congestion problems that are currently developing in the U.S. national air transportation system. Forecasts indicate that air traffic congestion is likely to worsen in the future. Although some of this congestion may be arising due to inadequate capacity, there is a consensus within the aviation community that some of these problems can be addressed through better airspace management.[3] Research initiatives are currently underway to develop traffic analysis and management tools that will lead to a more efficient management of airspace resources. Development of high-fidelity simulation models of the airspace is an essential component of this research effort. The Future ATM Concepts Evaluation Tool (FACET)[4] and its derivatives[5] are examples of such efforts. These tools simulate the motion of every aircraft in the airspace system by the use of flight plans, aircraft performance models, and wind forecasts.

Although it is necessary to use aircraft-level models of the airspace for certain ATM applications such as conflict detection and resolution, these detailed models are not easily amenable to analytical treatment for solving traffic flow management problems. This is largely due to the dimension of the problem because the dynamic representation of each aircraft in the environment requires at least three differential equations per aircraft. Hence, it is desirable to develop dynamic models of the airspace that are of lower order and represent the system dynamics with acceptable fidelity for the problem at hand. This can be accomplished by pursuing modeling approaches whose order depends on the spatial complexity of the air traffic environment, rather than on the number of aircraft in it. This approach to modeling can be termed the Eulerian[6] approach. The Eulerian modeling approach has been highly successful in disciplines such as fluid mechanics and heat transfer.

The focus of the research discussed in this paper is on modeling, analysis, and control of air traffic flows with the Eulerian approach. The Eulerian modeling technique spatially aggregates air traffic to generate models of air traffic flow in one-dimensional control volumes. Consequently, the order of the airspace model depends only on the number of control volumes used to represent the air traffic environment and not on the number of aircraft operating in them. Note that in such a modeling approach, the operational details of individual aircraft are lost, and only the aggregate properties of aircraft operating in each of the control volumes are preserved. However, this is appropriate for solving traffic flow management problems. The present approach has its basis in road traffic[7–11] modeling techniques and will be termed the Eulerian air traffic flow model.

The fidelity of the present model depends on the air traffic flow rate variations and the spatial resolution of the discretized airspace. Discussions on the formulation of the Eulerian model and its fidelity assessment will be given in Sec. II. If the speed of air traffic can be considered to be a constant within each control volume, the Eulerian model yields linear, discrete-time difference equations. Algorithms from computational linear algebra[12,13] can be used to develop reliable software for the manipulation and simulation of these models.

The theory of linear discrete-time dynamic systems[14] can be employed in conjunction with the Eulerian traffic flow models to derive a variety of useful analytical results. For instance, the controllability of the airspace with respect to flow and departure controls at various locations in the airspace can be assessed with a well-conditioned numerical algorithm. As another example, matrix manipulations can be used to derive discrete-time transfer functions[14] from the Eulerian model to determine the traffic latency between any points of interest in the airspace. The latency can then be used to determine the efficacy of various candidate control strategies to manage the flow of traffic.

The Eulerian model can also be used to derive information about the stability and robustness of the airspace under various flow control strategies. If the control strategies are nonlinear, computational approaches to Lyapunov stability theory[15,16] can be used to derive useful results. In addition to being useful for traffic flow analysis, the Eulerian model can be used to synthesize flow control schemes and flow observer algorithms. Linear multivariable control theory[14] provides a variety of design methodologies that can use the Eulerian traffic flow model to derive stable and robust flow control algorithms. If the system is completely controllable, and if all of the states of the model are available from measurements, control laws can be designed to provide desired characteristics to the closed-loop system.

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These algorithms could be used either as decision aids for manual control, or as the basis for automated airspace flow control. Linear control theory can also be used to identify the need for introducing feedback loops in the air traffic environment to make it amenable to human control. The conclusions from the present research are given in Sec. III.

II. Modeling, Analysis, and Control of Air Traffic Flow

The objective of air traffic flow control is to synthesize arrival, departure, and en route strategies that will result in a smooth flow of air traffic through the national airspace system (NAS). The control process achieves these objectives by metering arrivals and/or delaying departures from various airports, and/or by adjusting the traffic speed and spacing in certain regions of the airspace. If the traffic demand is low most of the time, effective flow control can be achieved manually. However, as the traffic demand increases, purely manual approaches may produce undesirable flow fluctuations, resulting in inefficient utilization of the airspace. Analytical tools for flow analysis and control may then become essential.

Research discussed in this section advances a new approach to modeling, analysis, and control of air traffic flow. The approach exploits classical road traffic flow modeling methods reported in the literature,[7–11] together with modern control theory, to derive useful flow control and analysis methodologies.

A central component of the research presented in this section deals with the modeling of air traffic by spatially aggregating the dynamic behavior of a group of aircraft in a region in space. Under physically reasonable assumptions, these models are in the form of linear, discrete-time dynamic systems, which can be aggregated by the use of block diagram algebra[14] to model any complex traffic flow pattern. Because of the linear nature of the model, it can also be used to propagate stochastic properties of traffic flow between any pair of points in the airspace, thereby providing important statistical measures for air traffic flow control. The modeling process will be described in further detail in Sec. II.A. The analysis methodology will be discussed in Sec. II.B.

An important use of the air traffic flow model is in enabling the design of air traffic flow control strategies to achieve desired dynamic properties for the airspace. For instance, arrival/departure flow control strategies at appropriate airports can be designed to achieve efficient flow. The flow model can be employed for synthesizing control strategies that are robust with respect to disturbances introduced by weather or any other factors influencing the air traffic environment. A systematic approach for the synthesis of flow control strategies will be discussed in Sec. II.C.

A. Modeling Air Traffic Flow

Air traffic flow can be modeled by integration of the equations of motion for every aircraft in the airspace by the use of their individual flight plans. This approach is employed in air traffic simulation tools such as FACET. The number of dynamic equations in such an approach is directly proportional to the number of aircraft in the environment.

Because the traffic flow control problem is more concerned with the aggregate properties of groups of aircraft rather than the dynamic behavior of individual aircraft, an approach that combines the dynamic properties of multiple aircraft may be more efficient in practical applications. This aggregation can be accomplished by division of the airspace into a number of interconnected regions and then describing traffic flow dynamics in terms of the flow properties in these regions. For instance, the rate of change of the number of aircraft in a region can be described based on the traffic flowing into and out of the region. Constitutive laws such as the conservation of the number of aircraft in the airspace can be invoked to formulate the equations describing the dynamics of air traffic flow in the region.

Such an approach to air traffic flow modeling can be termed the Eulerian[6] approach, in contrast to the more obvious Lagrangian approach, which models the trajectories of every individual aircraft in the environment. Because the Eulerian model describes the air traffic flow dynamics in regions of the airspace, the model order is independent of the number of aircraft in the airspace. Thus, in contrast with the Lagrangian approach, the efficiency of the Eulerian description of the air traffic flow dynamics improves with the number of aircraft in the environment.

Note that, unlike the Lagrangian modeling technique, the Eulerian approach does not preserve the identity of individual aircraft in the air traffic environment. A separate bookkeeping procedure will have to be devised to keep track of the location of individual aircraft in the environment. Because the Eulerian traffic flow models will eventually be used in conjunction with airspace simulation tools such as FACET, this will not limit the usefulness of the present modeling approach. It will be shown in the following sections that the Eulerian model of air traffic flow can deliver analysis and design information useful for air traffic flow control.

1. Eulerian Traffic Flow Model

Formulation of models for road traffic by the use of fluid-mechanical aggregation concepts has been of interest since the 1950s. The seminal papers in this area were written by Lighthill and Whitham[7] and Richards[8] (LWR) whose approaches are collectively known as the LWR theory in the road transportation literature. The LWR theory asserts that the relation between the flow rate q and linear density ρ may vary as a function of location, but not as a function of time,[9] that is,

$$\rho(x, t) = f[q(x, t), x] \tag{1}$$

$$q(x, t) = g[\rho(x, t), x] \tag{2}$$

for some given functions $f(\,)$ and $g(\,)$. With no entering or exiting traffic, the conservation equation according to LWR theory produces a one-dimensional partial differential equation of the form[9]

$$\frac{\partial q(x, t)}{\partial x} = -\frac{\partial \rho(x, t)}{\partial t} \tag{3}$$

The traffic flowing into the control volume modulates the density of the control volume and thereby changes the outflow from the control volume. The LWR theory is often referred to as the hydrodynamic theory of traffic flow due to its similarity to fluid flow physics.

Although the LWR partial differential equation can be solved by the use of the method of characteristics,[9] the solution process can become extremely involved whenever the system experiences rapid spatial density changes or shocks. It has recently been shown[10,11] that a special, discretized form of the LWR conservation equation can provide solutions that remain valid even in the presence of shocks. This discretization process produces a system of interconnected, one-dimensional control volumes with temporal difference equations describing the linear flow dynamics in each control volume. Any air traffic environment can be modeled by a system of such interconnected control volumes.

Because the objective of the present research is to analyze and synthesize flow control strategies, the Eulerian model will need to include the effects of air traffic control actions. Moreover, the model will be cast in terms of flow rates and the number of aircraft in the control volume. The merging and diverging of traffic streams will also be handled differently from the methods of Ref. 11. These changes produce models that are different from those used in road traffic simulation studies.

Figure 1 illustrates the concept of a control volume in the context of air traffic. The control volume is a one-dimensional entity of a specified length, with aircraft entering at its input and leaving at its output. The air traffic control actions modulate the outflow from the control volume by varying the speeds or by stretching the paths of aircraft inside the control volume. In the interest of maintaining transparency in the modeling process, it will be assumed that the aircraft speed at the input and output of the control volume are equal. This approach will provide crucial simplifications in the model. Additional clarifications on the modeling of the air traffic control actions in the model will be provided in the following paragraphs.

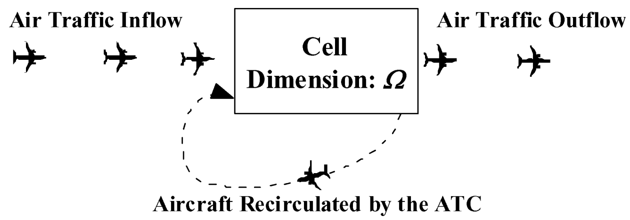

Fig. 1 Model of one-dimensional traffic flow.

Let p_j be the number of aircraft in the control volume j at the time instant i. Then the change in the number of aircraft in this control volume can be described by the discrete-time difference equation,

$$p_j(i+1) = p_j(i) + \tau_j[q_{j-1}(i) - q_j(i)] \tag{4}$$

The number of aircraft entering the control volume j from the control volume $j-1$ in a unit interval of time is $q_{j-1}(i)$, and the number of aircraft leaving the control volume is $q_j(i)$. The time step τ_j is computed from the average aircraft speed in the control volume v_j and the control volume dimension Ω_j as $\tau_j = \Omega_j/v_j$. Thus, τ_j is the time that an aircraft takes to transit through the control volume. Additionally, the following relationships can be obtained from inspection: 1) spatial density of air traffic in the control volume, $\rho_j = p_j/\Omega_j$; 2) average spacing between aircraft in the control volume, $d_j = \Omega_j/p_j = 1/\rho_j$; and 3) relationship between flow rate, speed, and density, $q_j = \rho_j v_j = v_j/d_j$. Under normal conditions, the air traffic flow rate out of the control volume j will be proportional to the spatial density of air traffic and the average traffic speed:

$$q_j = v_j p_j/\Omega \tag{5}$$

To control flow through a control volume, the air traffic controller may vary the traffic speed or stretch the paths of the aircraft within the control volume. (In more extreme situations, some aircraft may be placed in holding patterns within the control volume.) Because the Eulerian model does not describe the behavior of individual aircraft, these effects can be lumped together by the introduction of an air traffic control flow rate q_j^{ATC} to modify the flow rate out of the control volume as

$$q_j = v_j p_j/\Omega - q_j^{\mathrm{ATC}} \tag{6}$$

To satisfy the conservation principle, this negative air traffic flow rate at the output can be added in as an additional inflow into the control volume. Physical limitations dictate an air traffic control (ATC) flow constraint of the form $0 \le \tau_j q_j^{\mathrm{ATC}} \le x_j$, that is, the flow out of a control volume in a time step cannot exceed the number of aircraft in the control volume.

With the foregoing discussions, the discrete-time difference equation for the jth control volume can be obtained as

$$p_j(i+1) = (1 - \alpha_j v_j \tau_j/\Omega_j)_j p_j(i) + \tau_j q_j^{\mathrm{ATC}}(i) + \tau_j q_{j-1}(i) \tag{7}$$

$$\tau_j = \Omega_j/v_j \tag{8}$$

$$q_j = v_j p_j/\Omega - q_j^{\mathrm{ATC}} \tag{9}$$

If the aircraft speeds inside the control volume are nearly constant, the coefficients of this difference equation can be considered to be constants.

In the cases where the aircraft are slowing down during descent from cruise conditions, or accelerating to cruise conditions during climb, control volume can be set up to define several constant speed segments to approximate the slowing down and speeding up of air traffic. Because the objective of the present research is to demonstrate the feasibility of the proposed modeling approach, this level of detail will not be included. In all that follows, the aircraft speed will be assumed more or less constant throughout the airspace under consideration.

With the foregoing, the Eulerian model may be recast in a more familiar form by definition of the number of aircraft in the control volume p_j as its state variable x_j, the outflow q_j as the control volume output y_j, and the ATC flow q_j^{ATC} as the control variable u_j. The Eulerian model is now in the form of a linear, discrete-time dynamic system of the form

$$x_j(i+1) = a_j x_j(i) + \tau_j u_j(i) + \tau_j y_{j-1}(i) \tag{10}$$

$$y_j(i) = b_j x_j(i) - u_j(i) \tag{11}$$

Note that the model is linear only if the ATC actions u_j are within the defined bounds. Otherwise, the model will contain a state-dependent control constraint of the form

$$0 \le \tau_j u_j(i) \le x_j(i) \tag{12}$$

The coefficients of the model are given in terms of the physical dimension of the control volume Ω_j and the aircraft speed through the control volume v_j as

$$a_j = (1 - v_j\tau_j/\Omega_j), \qquad b_j = v_j/\Omega_j, \qquad \tau_j = \Omega_j/v_j \tag{13}$$

The Eulerian model is now in a form suitable for analysis by the use of well-known techniques in modern control theory.

The next step is assessment of the fidelity of the Eulerian model. The Eulerian traffic flow model is an approximation of an air traffic environment. The number and distribution of the control volumes used to model a given air traffic environment depends on the desired modeling fidelity and computational efficiency. Typically, the physical dimensions of the control volumes depend on the expected speed of traffic and the desired time resolution. The time resolution necessary to reproduce a specific traffic flow pattern depends on the highest frequency of expected variations in the traffic flow. The well-known sampling theorem[14] dictates that the sample frequency should be at least twice the highest frequency in a signal. As an example, if the traffic flow data are expected to vary significantly over 30 min or more, the control volume model must have a time resolution of 15 min or less. If the average traffic speed is 400 kn (741 km/hr), this time resolution leads to a maximum control volume dimension of 100 n mile (185 km) (400 kn × 0.25 h). The airspace under consideration can be modeled by the use of one control volume of this dimension, or two or more control volumes of smaller dimensions.

Although the model accuracy improves with smaller control volume sizes, there is little or no benefit in making them too small. Because the minimum en route spacing between aircraft is 5 n mile, this represents the lower bound on the control volume dimension. With the minimum control volume dimension, only one aircraft would occupy a control volume at any given time instant. On the other hand, making the control volume dimension too large may result in computational errors introduced by the quantization processes inherent in the Eulerian model.

2. *Merge and Diverge Models*

In addition to one-dimensional traffic flow modeled by the control volumes as discussed in the preceding section, the airspace may contain points at which traffic from different directions may merge into single streams. There would also be points at which traffic from one stream diverges into two or more streams. These points will be referred to, respectively, as the merge nodes and diverge nodes in the present research. The models of the one-dimensional control volumes, together with the models for merge and diverge nodes, can be used to represent any air traffic environment. A conceptual merge node is given in Fig. 2.

The model of a merge node can be derived by invokation of the conservation principle. Because the node does not retain any aircraft as modeled in the present research, the number of aircraft arriving at the node must be equal to the number of aircraft leaving the node. Under the assumption that the aircraft speed remains invariant during the merge operation, the conservation principle implies that the aircraft flow rate out of the branch k must be equal to the sum of the air traffic flow rate arriving through branches $k-1$ and $k-2$. This leads to

$$q_k = q_{k-1} + q_{k-2} \tag{14}$$

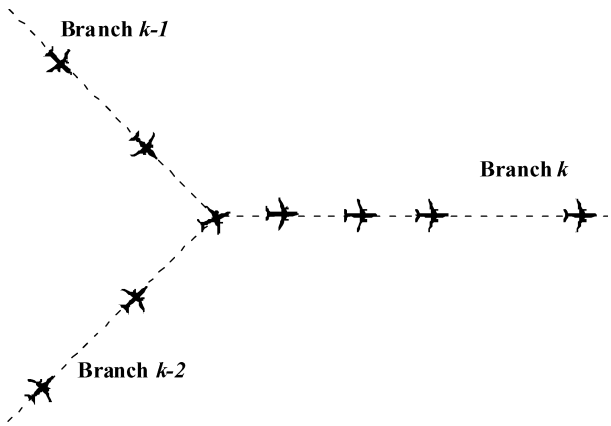

Fig. 2 Air traffic combining at a merge node.

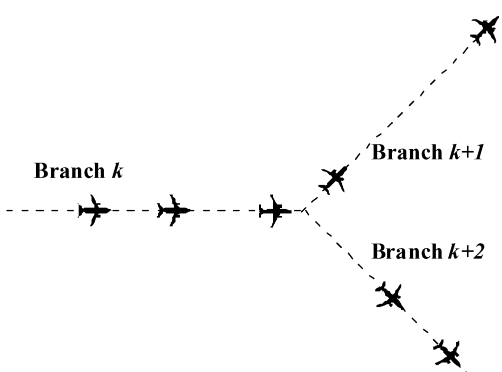

Fig. 3 Air traffic splitting at a diverge-node.

Note that this expression is the analogue of Kirchoff's current law from electrical circuit theory. Nodes with more than two incoming branches can be modeled as a cascade of two merge nodes.

Diverge-node models can be derived in an entirely analogous manner. A typical diverge node is shown in Fig. 3. The ratio of aircraft diverted to each of the outflow branches is defined as the divergence parameter β. Physical considerations yield a constraint on the divergence parameter as

$$0 \le \beta \le 1 \tag{15}$$

The traffic flow rate along each branch of the diverge node can be expressed in terms of the inflow into the node and the divergence parameter by the use of the conservation principle:

$$q_{k+1} = \beta q_k, \qquad q_{k+2} = (1-\beta) q_k \tag{16}$$

Merge and diverge models, in conjunction with the one-dimensional control volume models, can be used to model traffic flow pattern in any airspace. The next section will illustrate this process by the development of a flow model for an example air traffic environment. Models for merge and diverge nodes were derived in this section in terms of traffic flow rates. An alternate approach is to cast these models in terms of the number of aircraft arriving or leaving the node.

3. Eulerian Traffic Flow Model of an Example Airspace

Consider an environment composed of five airspaces containing four airports, with merging and diverging traffic streams as shown in Fig. 4. In this example, airports 1 and 2 only provide aircraft into the airspace (inflows), whereas the arrivals at airports 4 and 5 constitute system outflows. Airspace 4 handles overflight traffic as well as the traffic going to airport 4. For the purposes of the present modeling, it is assumed that three of these five airspaces have flow control capabilities.

The air traffic environment can next be decomposed into control volumes and merge and diverge nodes, as shown in Fig. 5. Note that the control volume dimension will determine the number of states

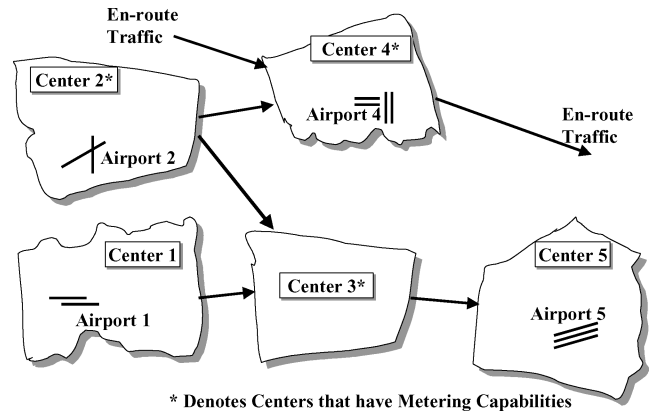

Fig. 4 Example air traffic environment.

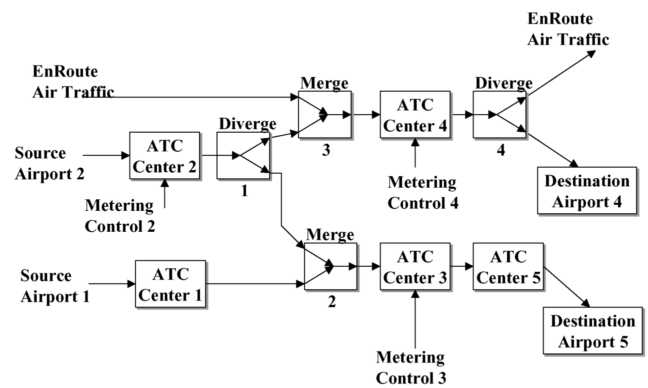

Fig. 5 Air traffic environment in terms of control volumes and merge and diverge models.

in each ATC center. For convenience, the average traffic speeds in all airspaces are assumed to be 400 kn, and the physical dimensions of each airspace are assumed to be integer multiples of 100 n mile. Airspaces 1, 2, and 5 are assumed to have a dimension of 100 n mile, and airspaces 3 and 4 are assumed to have a dimension of 200 n mile. The time step is assumed to be 0.25 h (one time unit). Thus, the airspaces 1, 2, and 3 will have one control volume each, and the airspaces 3 and 4 will have two control volumes each. Note that each control volume in the Eulerian model has one state equation and one output equation. The system dynamics can be written by inspection.

In the following equations, a superscript s denotes source or inflow quantities, superscript d denotes outflow quantities, and the subscript e denotes overflight traffic related quantities:

For airspace 1,

$$x_1(i+1) = a_1 x_1(i) + \tau[y_1^s(i), \qquad y_1(i) = b_1 x_1(i)]$$

For airspace 2,

$$x_2(i+1) = a_2 x_2(i) + \tau u_2(i) + \tau y_2^s(i)$$

$$y_2(i) = b_2 x_2(i) - u_2(i)$$

For airspace 3,

$$x_{31}(i+1) = a_{31} x_{31}(i) + \tau[y_1(i) + (1-\beta_1) y_2(i) + u_3(i)]$$

$$y_{31}(i) = b_{31} x_{31}(i) - u_3(i)$$

$$x_{32}(i+1) = a_{32} x_{32}(i) + \tau y_{31}(i), \qquad y_3(i) = b_{32} x_{32}(i)$$

For airspace 4,

$$x_{41}(i+1) = a_{41} x_{41}(i) + \tau\left[y_e^s(i) + \beta_1 y_2(i) + u_4(i)\right]$$

$$y_{41}(i) = b_{41} x_{41}(i) - u_4(i)$$

$$x_{42}(i+1) = a_{42}x_{42}(i) + \tau y_{41}(i), \qquad y_4(i) = b_{42}x_{42}(i)$$

$$y_4^d(i) = (1-\beta_4)y_4(i), \qquad y_e^d(i) = \beta_4 y_4(i)$$

For airspace 5,

$$x_5(i+1) = a_5x_5(i) + \tau y_3(i), \qquad y_5^d = y_5(i) = b_5x_5(i)$$

Note that airspaces 3 and 4 have been modeled with two state variables each, due to their physical dimensions.

The control variables in this model are the departure rates from airports 1 and 2, $y_1^s(i)$ and $y_2^s(i)$, respectively, and the flow controls in airspaces 2, 3, and 4, $u_2(i)$, $u_3(i)$, and $u_4(i)$, respectively. Not all of these control variables will be available at all time instants. For instance, operationally, it may be more desirable to use departure control rather than flow control to regulate traffic. Note that the aircraft inflow into the air traffic environment appears as a disturbance to the model whenever departure control is not used. The model states x_1, x_2, $x_{31}+x_{32}$, $x_{41}+x_{42}$, and x_5 are the number of aircraft in each of these centers at any time instant. The model outputs are the outflows, that is, arrivals, at airports 4 and 5 and the overflight traffic outflow $y_4^d(i)$, $y_5^d(i)$, and $y_e^d(i)$, respectively.

The preceding equations can be simplified and the state equation/output equation placed in the standard state-space[17] form. The dynamics of the air traffic environment can then be expressed in terms of the system matrix A, the control matrix B_1, the disturbance/control matrix B_2, the output matrix C, and the feedforward matrix D as

$$x_{i+1} = Ax_i + B_1u_i + B_2y_i^s, \qquad y_i^d = Cx_i + Du_i \tag{17}$$

Methods from linear system theory can now be used to analyze the system. Numerical values for the system parameters can be used to obtain a model suitable for further analysis. When the fundamental time unit is set as 0.25 h, with the flow proportionality parameter a_j being unity (unsaturated flow through the control volumes), the model parameters can be computed as $\tau = 1$, $a_j = 0$, $b_j = 1$, $j = 1, 2, \ldots, 5$.

For the present analysis, the divergence parameters are chosen to be $\beta_1 = 0.3$ and $\beta_4 = 0.8$. Note that the instantaneous values of the divergence parameters can be obtained from air traffic simulation tools such as FACET by the use of an averaging process.

B. Analysis of Air Traffic Flow Using the Eulerian Model

The linear, time-invariant dynamic model of the air traffic environment derived in the preceding section is in a form suitable for several different types of analyses. Some of these will be discussed in the following sections.

1. *Controllability*

Controllability analysis can be used to determine if a given air traffic environment can be controlled by the use of a specified combination of flow and departure control variables. Controllability analysis can be formulated to determine the minimal set of the flow and departure controls necessary to regulate the number of aircraft within desired limits in each airspace.

Controllability of a linear dynamic system can be assessed by examination of the rank of its controllability[14,17] matrix. Controllability tests can be conducted for various combinations of input variables. For instance, in the model given in Sec. II.A.3, if the three flow controls and airport 1 and 2 departure rates are included, the controllability matrix is a 7×35 matrix with a rank of 7. This denotes that the air traffic environment can be completely controlled with all of these inputs. The test can be conducted for various other combinations of input variables to determine the minimal set that will achieve full control of the air traffic environment. Such an analysis is presented in Table 1. The results in Table 1 were generated using the Control System Toolbox.[18]

It can be verified that the air traffic environment is not completely controllable with any one input. Bold-faced entries in Table 1 indicate that the air traffic environment is completely controllable by the use of departure rate control at the two airports, or through combinations of one departure rate control and one flow control. These represent the minimal set of inputs that can control the air traffic flow in the modeled environment.

Table 1 Controllability analysis of the air traffic environment

Metering control 2	Metering control 3	Metering control 4	Airport 1 departure rate	Airport 2 departure rate	Completely controllable?
Yes	Yes	Yes	Yes	Yes	Yes
Yes	Yes	Yes	Yes	——	Yes
Yes	Yes	Yes	——	——	No
Yes	Yes	——	——	——	No
Yes	Yes	Yes	——	Yes	No
Yes	Yes	——	Yes	——	Yes
——	——	——	**Yes**	**Yes**	**Yes**
Yes	——	——	**Yes**	——	**Yes**
——	Yes	——	Yes	——	No
Yes	——	——	——	Yes	No
——	Yes	——	——	Yes	No
——	——	Yes	Yes	——	No
——	——	Yes	——	Yes	No

Information provided by the controllability analysis can be extremely valuable for planning flow control strategies in the face of evolving traffic patterns. Controllability tests based on the Eulerian traffic flow model of the air traffic environment can be used as a decision aid for real-time flow control.

2. *Latency Analysis*

Latency analysis provides instantaneous dynamic relationships between any given pairs of points in the air traffic environment. These dynamic relationships describe how the changes in one of the system inputs affect the changes in system outputs, given that all other inputs are at their nominal values. These dynamic relationships can be termed the dynamic impedance to air traffic flow between two points of interest. In the terminology of control systems, these dynamic relationships are called the transfer functions.

For instance, it may be of interest to determine how the flow control policy at one of the ATC centers affects the traffic flow rate into one of the destination airports. Alternately, the central flow control may need to determine how the departure rate at one of the airports influences the throughput at the destination airport or how it affects the en route traffic. Latency analysis can provide quantitative insight into the manner in which dynamic variations at the inputs affect the outputs.

The Eulerian model of the air traffic environment can be used to compute instantaneous transfer functions between any desired input–output state pairs. Instantaneous transfer functions are computed around a nominal operating condition.

Let $\bar{x}_i$, $\bar{u}_i$, $\bar{y}_i^s$, and $\bar{y}_i^d$ denote the nominal values of the states, flow controls, source airport departure rates, and destination airport arrival rates, respectively. Dynamic equations describing the perturbations that surround this operating condition can be written as

$$(x_{i+1} - \bar{x}_{i+1}) = A(x_i - \bar{x}_i) + B_1(u_i - \bar{u}_i) + B_2\left(y_i^s - \bar{y}_i^s\right) \tag{18}$$

$$\left(y_i^d - \bar{y}_i^d\right) = C(x_i - \bar{x}_i) + D(u_i - \bar{u}_i) \tag{19}$$

or

$$\delta x_{i+1} = A\delta x_i + B_1\delta u_i + B_2\delta y_i^s, \qquad \delta y_i^d = C\delta x_i + D\delta u_i \tag{20}$$

Because the Eulerian model is a discrete-time model, transfer functions can be derived with the z transform. Transfer functions between inputs and the outputs can be derived by transforming the perturbation equations. In the interest of simplifying the notation, the z transform of the perturbation quantities will be denoted by uppercase letters hereafter. Application of z transforms to the system

Table 2 Latency analysis of the air traffic environment

Input	Destination airport 4	Destination airport 5	En route traffic outflow
Metering control 2	$-0.06(z^{-2} - z^{-3})$	$-0.7(z^{-3} - z^{-4})$	$-0.24(z^{-2} - z^{-3})$
Metering control 3	Gain = 0	$-1(z^{-2} - z^{-3})$	Gain = 0
Metering control 4	$-0.2(z^{-1} - z^{-2})$	Gain = 0	$-0.8(z^{-1} - z^{-2})$
Departure rate 1	Gain = 0	z^{-4}	Gain = 0
Departure rate 2	$0.06z^{-3}$	$0.7z^{-4}$	$0.24z^{-3}$

dynamics produces

$$Y^d(z) = \lfloor C(zI - A)^{-1}B_I + D\rfloor U(z) + \lfloor C(zI - A)^{-1}B_2\rfloor Y^s(z) \tag{21}$$

In the preceding matrix equation, I is an $n \times n$ identity matrix. The quantities within the brackets are transfer function matrices relating the air traffic environment input quantities to the output quantities. Transfer functions between any input–output pairs can be obtained by selecting out individual entries of these matrices. The transfer functions are normally given by the ratios of polynomials in z^{-1}. Software packages such as the Control System Toolbox[18] provide simple commands to compute the transfer functions from the matrices describing the system dynamics.

Application of latency analysis to the air traffic environment discussed in Sec. II.A.3 will result in a set of transfer functions as shown in Table 2. Although Table 2 gives the transfer functions between inputs and the three output variables of the model given in Sec. II.A.3, the development given in this section can be extended to provide transfer functions between the inputs and any other variables in the air traffic environment.

In Table 2, z^{-1} represents a unit time delay, 15 min in the present example. The transfer functions given in Table 2 can be used in a variety of ways. First, it can be used to determine how a certain input traffic pattern will influence the outputs. If the input traffic patterns can be defined in terms of well-known functions such as step, ramp, parabolic, or any other analytic function, the output traffic pattern can be computed analytically. In other cases, the transfer functions can be translated into simple recursive relationships and used to map the outputs in terms of the inputs.

As an example, the transfer function between departure flow rate at airport 1 and the destination airport 4 indicates that any action at the input will appear at the output after 4 time units or 1-h delay. Although the relationship between the traffic flow at departure airports and destination airports appear to be made up of pure time delays, the relationship between flow controls and the destination airports are more complex. Physical interpretation of these polynomial relationships will require explicit definition of the time histories of the flow controls. Note that the definition of closed-loop flow control policies for various inputs can make the transfer functions given in Table 2 much more complex.

An important use of the latency analysis is in the prediction of the statistical relationships between system inputs, states, and outputs. For instance, the transfer functions given in Table 2 can be used to predict the covariance of the traffic flow at the outputs in terms of the traffic flow at the inputs. Such information can be valuable for airspace planning and operational efficiency assessments.

Note that the latency analysis presented in this section ignored the traffic saturation conditions that may occur at isolated points in the air traffic environment. Traffic flow analysis, including saturation and other nonlinearities, will be of future interest.

3. *Stability Analysis*

Stability is an important property of a dynamic system. Although several definitions of stability are available in the literature, it can be conceptualized as the tendency of a dynamic system to return to equilibrium after an initial disturbance. Useful stability properties include bounded-input, bounded-output stability, and stability in a Lyapunov sense.[14,16,17] Although the air traffic environment may be nominally stable, introduction of specific flow control policies could sometimes result in marginally stable or unstable behavior. The performance of stability analysis on a periodic basis on the air traffic environment can reveal the potential for instability.

Because every aircraft that enters the environment will eventually leave the environment, without interference from external agents, the air traffic environment will always be globally asymptotically stable in the classical sense.[14] However, flow control procedures used to regulate traffic are based on feedback of the air traffic flow information. As is well known in control theory, even stable dynamic systems can become unstable under feedback. Moreover, systems that may be stable with very slow feedback loops can become unstable as the speed of feedback actions is increased. Instability may manifest itself in the air traffic environment as persistent flow oscillations at various points in the airspace. In many cases, it may be possible to modify the feedback loops algorithmically to stabilize a potentially unstable system. These factors make it important to study the stability of the air traffic environment. The objective of the stability analysis is to reveal the potential for these instabilities long before the airspace experiences any of its consequences.

For the Eulerian model of the example airspace, all seven eigenvalues of the system matrix are located at the origin in the complex z domain, which shows that the airspace is stable. Note that this conclusion also agrees with the physical intuition. Physically, the flow instabilities can occur due to the fact that some of the flow control logic used in practice may not explicitly account for the time delays in the propagation of the inflow.

C. Air Traffic Flow Control System Design

An important use of the Eulerian traffic flow model is in the synthesization of algorithms for air traffic flow control. When used with automatic control theory, Eulerian models provide a systematic approach for developing air traffic flow control strategies. Additionally, automatic control techniques can be used to introduce feedback loops in the air traffic environment to improve its dynamic properties to allow better manual control of the environment.

This section will illustrate how flow control systems can be synthesized with the Eulerian model. The model developed in Sec. II.A.3 has flow controls in three airspaces and the departure flow rates at airports 1 and 2 available to regulate the traffic. The control objective is to deliver aircraft to airports 4 and 5 at commanded rates in the presence of input flow variations. Overflight traffic is treated as an additional disturbance in the system.

To ensure tight control over the output variables, integral tracking error feedbacks[14] are introduced on the outflows at airports 4 and 5. With this, the open-loop system is of ninth order. From the output equation, it may be observed that the outflows at airports 4 and 5 are related algebraically to the model states x_{42} and x_5, respectively. Hence, in the interests of simplification of the control law, the flow rate commands are cast in terms of commands to these state variables. The flow control law has the form

$$\begin{bmatrix} u_2(i) \\ u_3(i) \\ u_4(i) \\ y_1^s(i) \\ y_2^s(i) \end{bmatrix} = K \begin{bmatrix} x_1(i) - x_1(0) \\ x_2(i) - x_2(0) \\ x_{31}(i) - x_{31}(0) \\ x_{32}(i) - x_{32}(0) \\ x_{41}(i) - x_{41}(0) \\ x_{42}(i) - 5y_4^c \\ x_5(i) - y_5^c \\ \sum x_{42}(i) - 5y_4^c \\ \sum x_5(i) - y_5^c \end{bmatrix} \tag{22}$$

The feedback control gain K is obtained the with linear quadratic regulator (LQR) theory.[14] Weighting matrices on the states and controls were chosen as identity matrices. The MATLAB® Control System Toolbox[18] was used for the design.

The performance of the closed-loop flow and departure control system is next evaluated in a simulation. The initial conditions used in the simulation were

$$[3 \quad 10 \quad 10 \quad 10 \quad 10 \quad 10 \quad 10]^T$$

The initial conditions for both integrators were set to zero. The inflow rate from airport 1 was set at a fixed value of 3 aircraft/time unit, and the inflow rate from airport 2 was set at 10 aircraft/time unit. Seven overflight aircraft were assumed to enter the airspace during every time unit. The control commands were to maintain the arrival rate at airport 5 at 10 aircraft per time unit, and the arrival rate at airport 4 at 2 aircraft/time unit.

For the simulations, limits were placed on the flow controls so that they could not go below zero and also on the sums of the departure controls with their corresponding source flows, so that the inflow rates could not go below zero.

Figures 6–8 show the response of the closed-loop system for a sinusoidally varying flow from source airport 2. The inflow had a mean of 10 aircraft/time-unit and an amplitude of 3 and is plotted in Fig. 8. The initial conditions and the inflows from source airport 1 and the overflight traffic were as specified earlier. From Fig. 7, note that the flow rates out are well regulated, although there are some oscillations during the downswing in the inflow.

The results given in this section illustrate how linear discrete-time control theory can be used to synthesize flow control logic with the Eulerian air traffic flow model. Although reasonable results were obtained by the imposition of control constraints and quantization, this was done after the linear controller was designed. Control system design methods that can directly take into account these types of constraints will be of future interest.

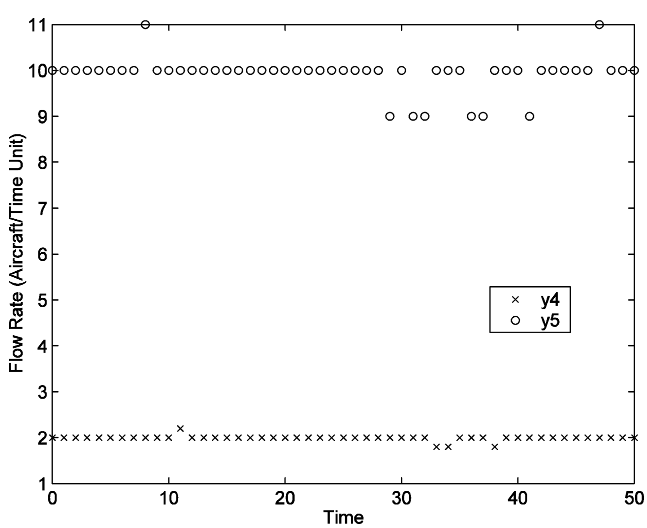

Fig. 6 Flow rates out of airports 4 and 5 vs time, seven-control volume model with LQR and control limits, sinusoidal inflow at source airport 2.

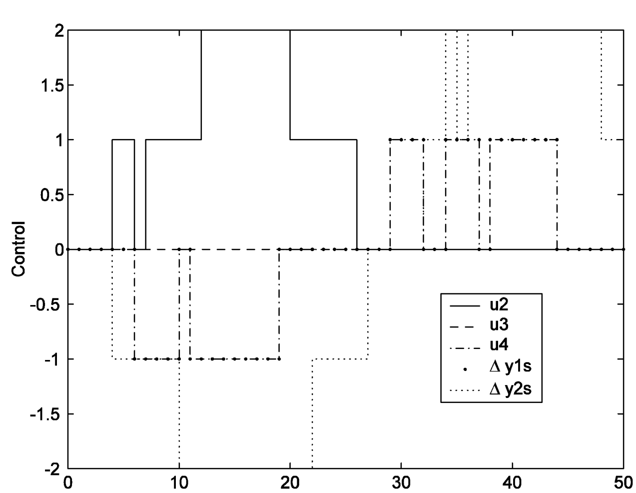

Fig. 7 Control signals vs time, seven-control volume model with LQR and control limits, sinusoidal inflow at source airport 2.

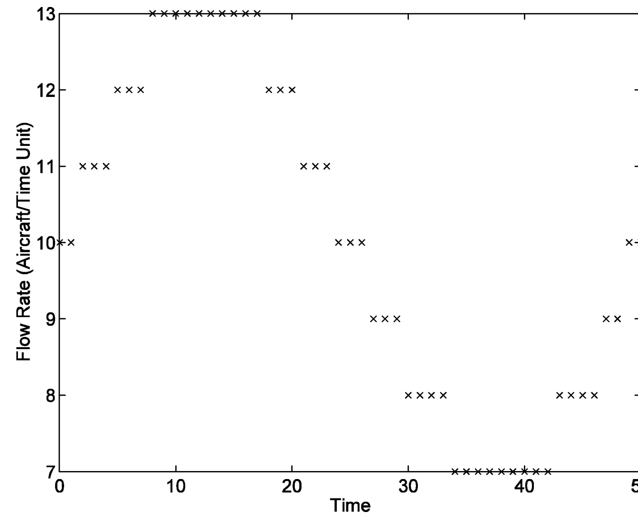

Fig. 8 Source airport 2 flow rate vs time.

III. Conclusions

This paper advanced a new approach for modeling the air traffic environment by the use of the Eulerian approach and demonstrated its use in carrying out various types of air traffic flow analyses and the synthesis of traffic flow control schemes. The Eulerian approach provides a systematic technique for spatial aggregation of air traffic and can be used to model complex air traffic patterns. The flow control scheme advanced in this paper can either be used in an automatic manner, or it can be used as a decision aid in flow control operations. The analysis and synthesis methodologies discussed in this paper can also be used to improve the condition of the air traffic environment by placing appropriate feedback loops to make the dynamics amenable to human control.

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